# PAPR Reduction in OFDM Systems Using A Piecewise Linear Companding Transform on Cyclic Shift PTS

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**Abstract – Orthogonal frequency division multiplexing (OFDM) signals have high peak-to-Average power ratio (PAPR). PAPR is a major drawback in the application of OFDM communication. High PAPR, which causes distortion when OFDM signal passes through a nonlinear high power amplifier and reduce system efficiency. We have various methods for the reduction of PAPR, Among various methods Partial Transmit Sequence (PTS) is best method. A cyclic shifted sequences (CSS) scheme is evolved from the PTS scheme to improve the PAPR reduction performance. In the CSS scheme challenging one is the shift value (SV) set should be carefully selected because those are closely related to the PAPR reduction performance of the CSS scheme. However, as Companding transform is an extra operation after the modulation of OFDM signals, companding schemes reduce PAPR at the expense of increasing the Bit Error Rate (BER). In this paper , a new piecewise linear companding transform is proposed aiming at mitigating companding distortion. With the careful design of the companded peak amplitude and the linear transform scale, the proposed transform can achieve better reducing PAPR effectively.**

**Index Terms – Orthogonal Frequency Division Multiplexing (OFDM), Peak-to-Average Power Ratio (PAPR), Partial Transmit Sequence (PTS), Cyclic Shift Sequences (CSS), Complementary Cumulative distribution Function (CCDF), Piecewise Linear Companding Transform.**

## 1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a multicarrier modulation method utilizing the orthogonality of subcarriers. Multi carrier modulation is a thought of parallel transmission having several parallel channels between the transmitter and receiver. OFDM systems suffers from high peak-to-average power ratio (PAPR) of the transmit signal. OFDM has been adopted as a standard modulation method in many wireless communication systems such as digital audio broadcasting (DAB), digital video broadcasting (DVB), IEEE 802.11 wireless local area network (WLAN), and IEEE 802.16 wireless metropolitan area network (WMAN). Similar to other multicarrier schemes, OFDM has a high peak-to-average power ratio (PAPR) problem, which makes its implementation quite costly. Over the last few decades, various schemes to reduce the PAPR of OFDM signal sequences have been proposed such as clipping, coding, active constellation extension (ACE) [2], tone reservation (TR), partial transmit sequence (PTS) [3], and selected mapping (SLM) [4], [5].

The PTS scheme, Like the SLM scheme statistically improves the PAPR of OFDM signals without signal distortion. In the PTS scheme, the input symbol sequence is partitioned into a number of disjoint input symbol subsequences. Inverse fast Fourier transform (IFFT) is then applied to each input symbol subsequence and the resulting OFDM signal subsequences are combined after being multiplied by a set of rotation factors. Next the PAPR is computed for each resulting sequence and then the OFDM signal sequence with the minimum PAPR is transmitted. The cyclic shift sequence (CSS) scheme is better than the PTS scheme from every aspect. First, its PAPR reduction performance is better than the PTS scheme's [4]. Even though the authors in [4] analyzed the Class-III SLM scheme, the Class-III SLM scheme can be viewed as a combination of the PTS and CSS schemes, where it uses both cyclic shift and multiplication of rotation factors to the OFDM signal subsequences as in [10]. The CSS scheme shows a better PAPR reduction performance than the PTS scheme's when they have the same computational complexity (the number of subblocks) and use the same number of alternative OFDM signal sequences. In this paper, we investigate how to select the shift value (SV) sets[1] and companding peak amplitude in order to boost the PAPR reduction performance of the CSS scheme. We introduce some criteria to select the good SV sets considering the autocorrelation function (ACF) of OFDM signal subsequences, and then verify its validness through simulations. This paper is as follows, Section2 explains Preliminaries. Section3 explains select shift value (SV) sets in the CSS scheme. Section4 explains about proposed scheme. Section5 explains the simulation results and finally Section6 explains Conclusion.

## 2. PRELIMINARIES

- OFDM System and PAPR
- Cyclic Shifted Sequences (CSS)

### 2.1. OFDM System and PAPR

In an OFDM system, IFFT generated an OFDM signal sequence in time domain as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(K) e^{j\frac{2\pi kn}{N}} \quad (1)$$

Where N is the number of subcarriers, X = {X(0), X(1),…….,X(N-1)} is an input symbol sequence in frequency domain, and

x = {x(0), x(1),……,x(N-1)} is an OFDM signal sequence in time domain. The PAPR of the OFDM signal sequence x is defined as

$$\text{PAPR}=\frac{max_{0\leq n<N}|x(n)|^2}{E[|x(n)|^2]} \quad (2)$$

Where E[ • ] represents the expectation.

### 2.2. Cyclic Shifted Sequences (CSS)

Figure 1, shows a block diagram of the CSS scheme testing $U$ alter-native OFDM signal sequences in total [6]. In the CSS scheme, $X$ is divided by a certain partitioning pattern into $V$ disjoint sub-blocks, input symbol subsequences $X_1$, $X_2$,…….., $X_V$. Then IFFT converts the $V$ sub blocks in frequency domain to the $V$ OFDM signal subsequences in time domain $x_1$, $x_2$,……, $x_V$, where $x_v$ = {$x_v(0)$, $x_v(1)$, …….,$x_v(N-1)$}, $1\leq v \leq V$. For simplicity, we assume that both $N$ and $V$ are integers of power of two. After that, the $V$ OFDM signal subsequences are cyclically shifted and combined together to make the $u$-th ($1\leq u \leq U$) alternative OFDM signal sequence as

$$x^u = \sum_{v=1}^{V} x_v^u \quad (3)$$

Where $x_v^u$ denotes the leftward cyclically shifted version of $x_v$ by some integer $\tau_v^u$ ($1\leq v \leq V$). That is,

$$x_v^u = \{x_v(\tau_v^u), x_v(\tau_v^u+1),\ldots, x_v(N-1), x_v(0),\ldots, x_v(\tau_v^u-1)\}. \quad (4)$$

As the SLM or PTS schemes, the candidate with the lowest PAPR, is chosen by exhaustive search for transmission with $[log_2 U]$ bits side information. Some additional techniques at the receiver using side information can be recovered [8]. Like the PTS scheme, the CSS scheme can use three partition methods, i.e., random, adjacent, and interleaved partition methods. It is widely known that the random partition method gives the best PAPR reduction performance among them while the interleaved partition method gives the worst PAPR reduction performance but it needs the lowest computational complexity. The adjacent partition method is not meaningful practically because it needs a high computational complexity as the random partition needs, but it shows worse PAPR reduction performance than the random partition case. In this letter, we consider the three partition methods.

## 3. DESIRABLE SHIFT VALUE SETS

The true objective of the CSS scheme is to reduce the probability of PAPR exceeding some threshold level rather than to reduce the PAPR of each alternative OFDM signal sequence itself, we have U SV sets that make alternative OFDM signal sequences statistically independent as possible can perform well.

### 3.1. Without consideration of correlation of OFDM signal sequence

In fact, the components in an OFDM signal subsequence are not mutually independent, which will be shown in the next subsection. However for now, we assume that the components in the OFDM signal subsequences are mutually independent for simplicity. That is, we have

$$\text{E}[x_{v1}(n_1) \bullet \{x_{v2}(n_2)\}^*] = \begin{cases}\sigma^2, & v_1 = v_2 \text{ and } n_1 = n_2 \\ 0, & otherwise\end{cases} \quad (5)$$

Where $\sigma^2$ is a component power of an OFDM signal subsequence and { • }* represents the complex conjugate.

*Criterion 1*: Suppose that we have $U$ SV sets; For every (i , j) pair out of the U SV sets ($i \neq j$), the pair should satisfy the condition that the relative distances $\tau_v^i$ - $\tau_v^j$ mod N are distinct from each other for all v's.

Note that the *Criterion 1* is valid when the components in all alternative OFDM signal subsequences are mutually independent. However, actually the OFDM signal subsequence components are not mutually independent because the corresponding input symbol subsequences in frequency domain have $N - N/V$ zeros. That is, it is widely known that the independent input components make independent output components while they pass the IFFT. However, $N - N/V$ zeros means that there are correlations between input components, which causes the dependency between the output components.

### 3.2. ACF of OFDM signal subsequences

Let $S_v$ be the discrete power spectrum of the v-th OFDM signal subsequence $x_v$,namely,

$$S_v = \{ p(0), p(1), \ldots, p(N-1)\} \quad (6)$$

Where p(k) = E[$|X_v(k)|^2$], and *p(k)* can have the value of zero or one. This is due to the assumption that the modulation order of all subcarriers is equal and the average power is normalized to one. For example, if the interleaved partition is used, $S_1$ = {10101010} and $S_2$ = {01010101} when $N$ = 8 and $V$ = 2.

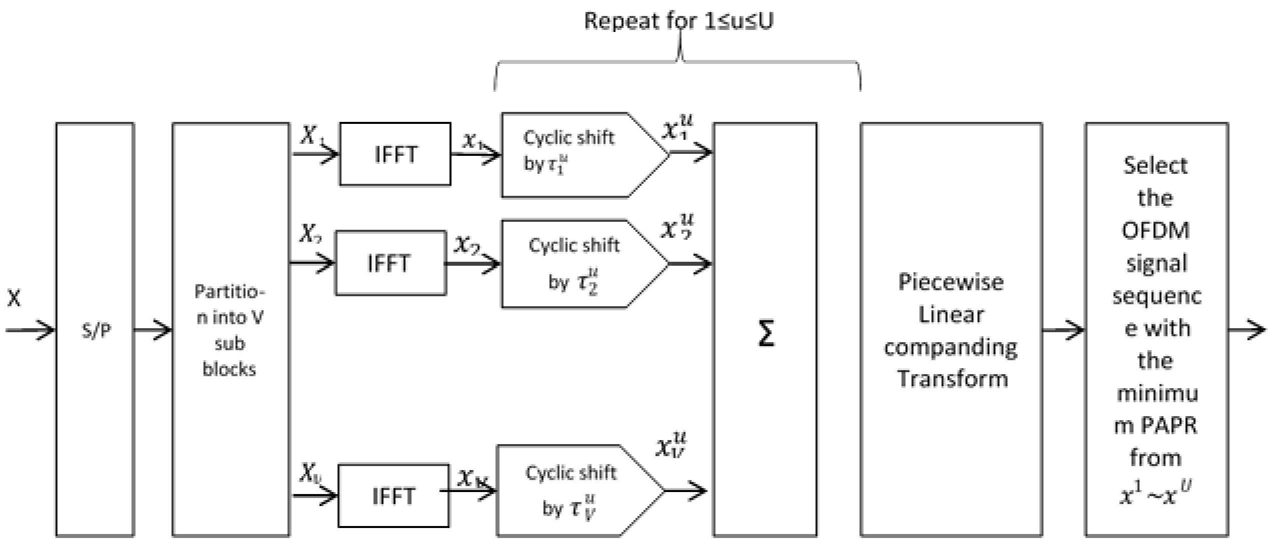


Figure 1 A block diagram of the companding transform on Cyclic Shift PTS

1) For Interleaved Partition: In this case, $S_v$ is an impulse train with an interval of V. Then, the ACF also becomes the impulse train as [11]

$$\left|R_{x_v}(m)\right| = \begin{cases} \frac{\sqrt{N}}{V} & \text{if } m = 0 \mod \frac{N}{V} \\ 0 & otherwise. \end{cases} \qquad (7)$$

2) For Adjacent Partition: In this case, $S_v$ is a rectangular function with a width of N/V. Then the ACF becomes the function as [11]

$$\left|R_{x_v}(m)\right| = \begin{cases} \frac{\sqrt{N}}{V} & if\ m = 0 \\ \frac{\sin(m\pi/V)}{\sqrt{N}\sin(m\pi/N)} & if\ m \neq 0. \end{cases} \qquad (8)$$

3) For Random Partition: In this case, $S_v$ can be viewed as a binary pseudo random sequence. Then the ACF has a shape similar to a delta function, where the components except $m = 0$ are close to zero.

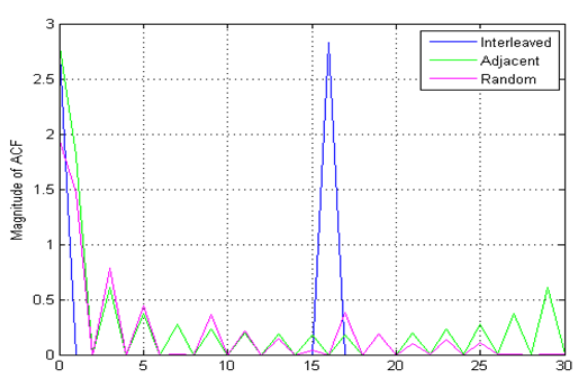


Figure 2 Magnitude of ACFs for different partition cases

Figure 2. shows an example of the magnitudes of ACFs corresponding to the following power spectrum when N = 32 and V = 2; $S_1$ = {1010⋯⋯1010} for an interleaved partition; $S_1$ = {11⋯1100⋯00} for an adjacent partition; $S_1$ = {10010110011111000110111010100000} for a random partition, which is an one zero padded m-sequence with length 31; clearly, $S_2$ is a complement of $S_1$ in each partition case, and the shapes of $\left|R_{x_v}(m)\right|$ for v=1 and v=2 are same.

3.3. Desirable Shift Value Sets With Consideration of ACF of OFDM Signal Subsequences

Now we investigate the desirable SV sets with consideration of ACF of the OFDM signal subsequence for three partition cases.

1) For Random Partition: In this case, the shape of the ACF is similar to a delta function. Therefore, the *Criterion 1* can be valid criterion.

*2) For Interleaved Partition:* The impulse train ACF in (14) means that components in the OFDM signal subsequence are related to each other as

$$E[x_{v1}(n_1) \bullet \{x_{v2}(n_2)\}^*] = \begin{cases} \sigma^2, v_1 = v_2\ and\ n_1 = n_2\ mod\ \frac{N}{V} \\ 0, & otherwise \end{cases} \qquad (9)$$

Therefore, *Criterion 1* has to be slightly modified as follows.

*Criterion 2*: Suppose that we have *U* SV sets; For every (i , j) pair out of the U SV sets (i ≠ *j*), the pair should satisfy the condition that the relative distances $\tau_v^i$ - $\tau_v^j$ mod N/V are distinct from each other for all v's.

3) For Adjacent Partition: Like the proofs of *Criterion 1* and *Criterion 2*, we may also derive the optimal

condition of the *U* SV sets in this case. However, it may be very complicated, which is not the simple case with zero or one. Therefore, we give a rough criterion for the adjacent partition case based on the rough interpretation.

*Criterion 3*: Suppose that we have *U* SV sets; For every (i , j) pair out of the U SV sets ($i \neq j$), the pair should satisfy the condition that the relative distances $\tau_v^i$ - $\tau_v^j$ mod N are distinct from each other for all v's. Furthermore, the mutual differences of the *V* relative distances ($\tau_1^i - \tau_v^j$, $\tau_2^i - \tau_2^j$,………, $\tau_V^i - \tau_V^j$ mod N) should be as close to N/2 as possible.

## 4. LINEAR COMPANDING TRANSFORM

Piecewise linear companding transform is proposed in this section. Then, based on the theoretical analysis Transform parameters are carefully designed. When the original signal $x_n$ is companded with a given peak amplitude $A_c$, the proposed companding scheme shown in Figure 3, clips the signals with amplitudes over $A_c$ for peak power reduction, and linearly transforms the signals with amplitudes close to $A_c$ for power compensation. Then, the companding function of the proposed companding scheme is

$$\text{h(x)} = \begin{cases} x & |x| \le A_i \\ kx + (1-k)A_c & A_i < |x| \le A_c \\ sgn(x)A_c & |x| > A_c \end{cases} \quad (10)$$

where *sgn(x)* is the sign function.

Consequently, the decompanding function at the receiver is

$$h^{-1}(x) = \begin{cases} x & |x| \le A_i \\ (x-(1-k)A_c)/k & (1-k)A_c < |x| \le A_c \\ sgn(x)A_c & |x| > A_c \end{cases} \quad (11)$$

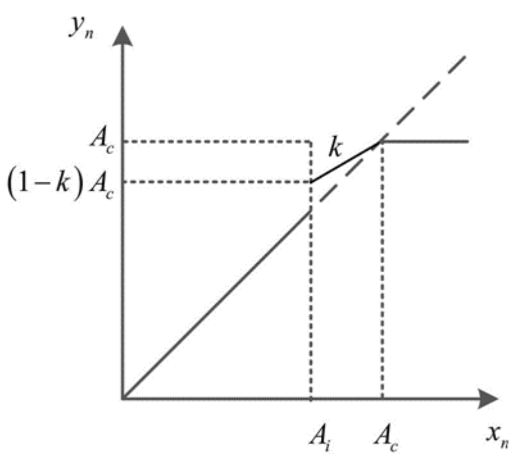


Figure 3 Linear companding transform

It is obvious that the proposed companding transform is specified by parameters $A_c$, $A_i$ and *k*. $A_c$ is the peak amplitude of the companded signals. As the average signal power is maintained constant, then according to the definition of PAPR in (2) the PAPR value of the proposed scheme that can be achieved theoretically is determined by $A_c$. With a preset theoretical PAPR value, $A_c$ can be determined as $A_c = \sigma_x 10^{PAPR_{preset}/20}$. With determined $A_c$, parameters $A_i$ and k can be obtained by solving in the paper [9].

## 5. RESULTS AND DISCUSSIONS

### 5.1. Comparison of partition techniques

Prior verifying our proposed criteria for the Piecewise linear companding on CSS scheme. The CSS scheme uses the well designed U SV sets satisfying *Criterion 1* and *Criterion 2*. Now, to verify the above proposed criteria for the CSS scheme are valid, we construct the *U* SV sets in two different ways. That is, the solid lines in Figure 4, show the PAPR reduction performance of the case when the *U* SV sets satisfy the above criteria well. On the other hand, the dotted lines in Figure 4, show the PAPR reduction performance of the case that does not. In the simulations, we use *N* = 128, *U* = 4, and *V* = 4 in common. The 16-QAM is used for all following simulations.

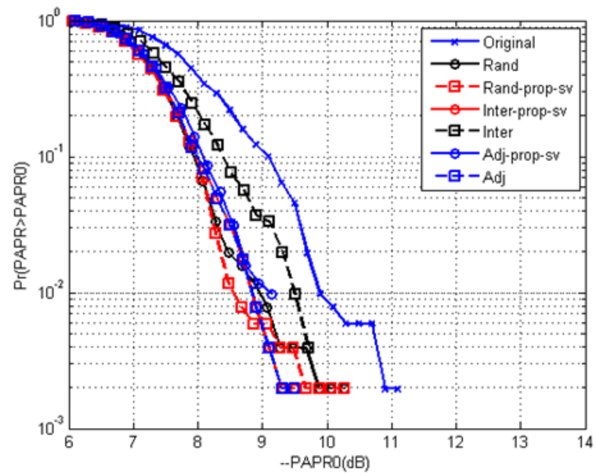


Figure 4 Comparison of the PAPR reduction performance of the CSS scheme for three partition cases, which are random, interleaved, and adjacent partition cases when N = 128, U = 4, and V = 4 according to the used SV sets.

### 5.2. Optimality of the criteria

It is hard to compare the PAPR reduction performance of the case using best SV sets to the cases using ALL possible SV sets through simulation because there are too many possible SV sets. Instead, in Fig. 5, we drew curve using randomly generated SV sets, where N=32, U=4, and V=4 are used. Also, we drew the curve using best SV sets satisfying *Criterion 2*.

The SV sets $\bar{\tau}^1$ = {1,1,1,1), $\bar{\tau}^2$ = {1,2,3,4), $\bar{\tau}^3$ = {1,3,5,7), $\bar{\tau}^4$ = (1,4,7,10) are used for the best SV sets. In Figure 5, we can observe the best PAPR reduction performance of the using SVsets.

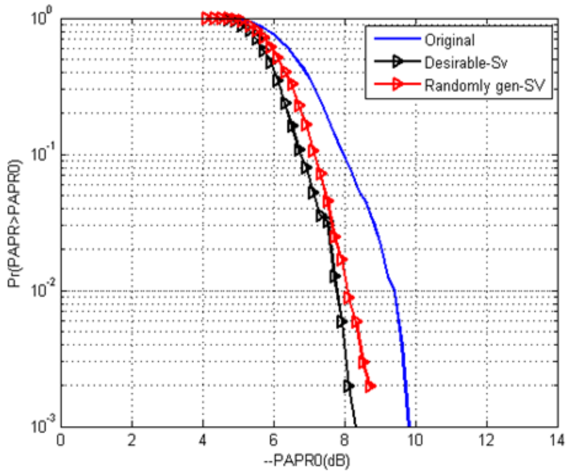


Figure 5 The optimality of the proposed SV sets when N=32, U=4, interleaved partition, and V=4 are used.

5.3. Proposed method

Prior to verifying our proposed method on CSS scheme, The comparison of the PAPR reduction performance between the CSS scheme without applying piecewise linear companding transform and CSS scheme with piecewise linear companding transform. In Figure 6, we can observe the best PAPR reduction performance after applying piecewise linear companding transform on CSS.

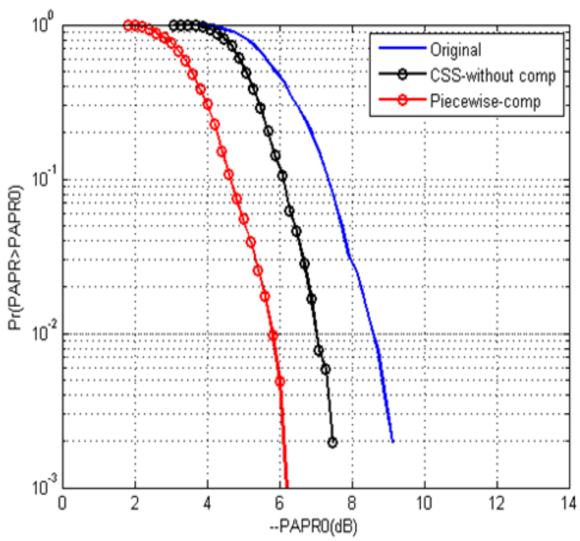


Figure 6 Comparison of the PAPR reduction performance of CSS with piecewise linear companding and CSS without piecewise linear companding

| METHODS / $PAPR_0$ (dB) | Original (CCDF) | CSS without Piecewise Companding (CCDF) | CSS with Piecewise Companding (CCDF) |
|---|---|---|---|
| 3.68 | 0.999 | 0.992 | 0.48 |
| 4.08 | 0.98 | 0.93 | 0.30 |
| 4.48 | 0.92 | 0.81 | 0.15 |
| 4.88 | 0.84 | 0.60 | 0.07 |
| 5.28 | 0.72 | 0.38 | 0.03 |
| 5.68 | 0.56 | 0.20 | 0.017 |

Table 1 Comparison between Original, CSS without Piecewise Linear Companding and with Piecewise Companding

Table 1, shows the PAPR values comparison between original OFDM signal sequence without any method apply, Cyclic Shift value PTS without Piecewise Linear Companding Transform and with applying Piecewise Linear Companding Transform . CSS with Piecewise linear companding transform improves the PAPR reduction.

## 6. CONCLUSION

The CSS scheme is the very popular and promising PAPR reduction scheme, which is evolved from the PTS scheme. In this letter, the criteria to select good SV sets are proposed, which can guarantee the optimal PAPR reduction performance of the CSS scheme. The criterion are proposed by considering the ACF of the OFDM signal subsequence for three different partition cases, random, interleaved, and adjacent partition cases. In the simulation results, the CSS scheme using the SV sets satisfying the proposed criteria shows better PAPR reduction performance than the case when the SV sets are not carefully designed.

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